2

X-552-71-495

PREPRINT

NASA TM X- 65920

ATS SIMULTANEOUS AND TURNAROUND RANGING EXPERIMENTS

JOHN S. WATSON
BARBARA H. PUTNEY

NOVEMBER 1971

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-552-71-495
Preprint

ATS SIMULTANEOUS AND TURNAROUND RANGING EXPERIMENTS

John S. Watson
Trajectory & Dynamics Branch

Barbara H. Putney
Geodynamics Branch

Trajectory Analysis & Geodynamics Division

November 1971

Goddard Space Flight Center
Greenbelt, Maryland

PRECEDING PAGE BLANK NOT FILMED

ATS SIMULTANEOUS AND TURNAROUND RANGING EXPERIMENTS

John S. Watson
Barbara H. Putney

ABSTRACT

This report explains the data reduction and spacecraft position determination used in conjunction with two ATS experiments – Trilateration and Turnaround Ranging – and describes in detail a multilateration program that is used for part of the data reduction process. The process described is for the determination of the inertial position of the satellite, and for formating input for related programs. In the trilateration procedure, a geometric determination of satellite position is made from near simultaneous range measurements made by three different tracking stations. Turnaround ranging involves two stations; one, the master station, transmits the signal to the satellite and the satellite retransmits the signal to the slave station which turns the signal around to the satellite which in turn retransmits the signal to the master station. The results of the satellite position computations using the multilateration program are compared to results of other position determination programs used at Goddard. All programs give nearly the same results which indicates that because of its simplicity and computational speed the trilateration technique is useful in obtaining spacecraft positions for near synchronous satellites.

PRECEDING PAGE BLANK NOT FILMED

CONTENTS

		Page
Abstract		iii
I.	Introduction	1
II.	Data Types	3
III.	Algorithm and Formulas	3
IV.	Computation of the Inertial Coordinates of the Tracking Stations	9
V.	Computation of the Inertial Coordinates of the Satellite	11
VI.	Computation of the Longitude and Latitude of the Sub-Satellite Point	13
VII.	Results	15
VIII.	Conclusion	21
IX.	Acknowledgment	22
X.	References	22
Appendix		23

ATS SIMULTANEOUS AND TURNAROUND RANGING EXPERIMENTS

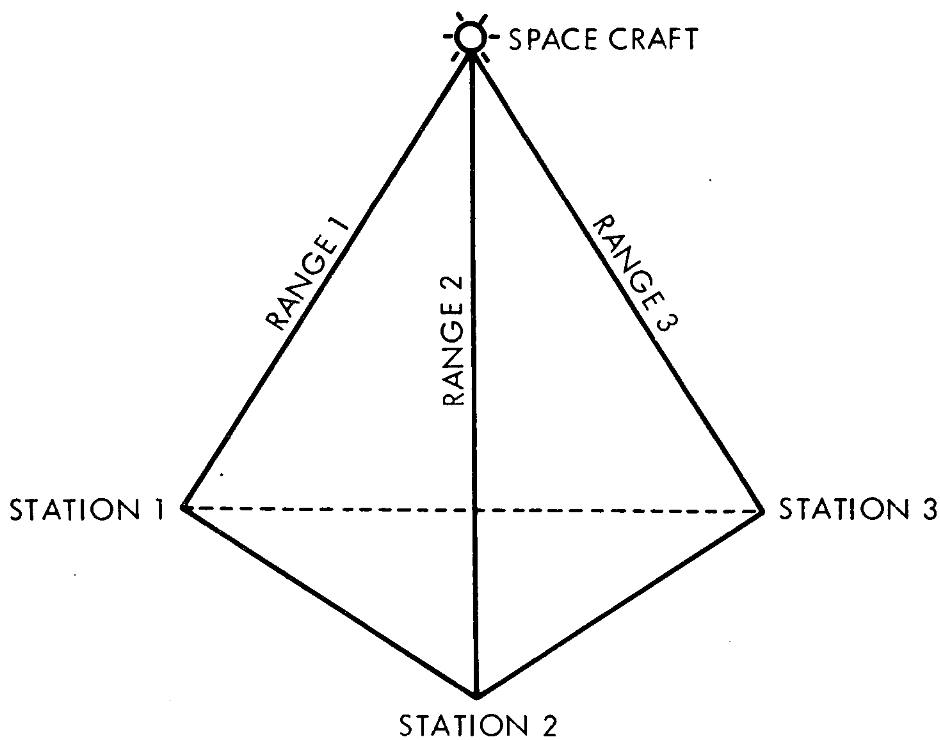
I. INTRODUCTION

The purpose of this document is to explain the data reduction used in conjunction with two ATS spacecraft position determination experiments – Trilateration and Turnaround Ranging – and to describe in detail one of the programs used for these experiments. In the trilateration procedure, a geometric determination of satellite position is made from near simultaneous range measurements made by three different tracking stations (see Figure 1). Turnaround ranging involves two stations; one, the master station transmits the signal to the satellite and the satellite retransmits the signal to the slave station which turns the signal around to the satellite which in turn retransmits the signal to the master station.

The Trilateration experiment used near simultaneous measurements of range to obtain a position vector of the satellite. In reality, three stations took measurements during the same ten minute period of time and by fitting this data with three polynomials as a function of raw data time one is able to create groups of simultaneous measurements from the polynomials. See page 18 for details of the ranging pattern. These groups of simultaneous measurements were then fed into a Hughes trilateration program – Trilatron 1 (Reference 1), and out of each group of measurements one position vector of the satellite was obtained. The advantage of the trilateration technique is that it does not require a sophisticated force model that is required by conventional orbit determination programs. The trilateration method requires that the three tracking stations be located so as to give a well determinated triangle, with the subsatellite point preferably being located in the triangle.

Turnaround Ranging uses a master station and some slave stations. The signal leaves the master station, travels to the satellite, returns to a slave station and is turned around to the satellite and then back to the master station, where it is recorded. By using this scheme, the slave station equipment need only be capable of turning the signal around to the satellite. Because the master station is tracking the satellite in its normal mode (two way delay time), the slave station range can be extracted from the four way delay time.

In addition to describing the Trilateration and Turnaround Ranging experiments and their results, this report describes a more general multilateration program to determine the position of a satellite in synchronous orbit. In this program the position of the satellite is computed by knowing the ranges from three or more observation points, and the times of the observations which should be as near the same time as possible. To obtain a solution, the program begins by making an initial estimate of the inertial coordinates of the satellite. For the first observation time, this initial estimate is obtained by using the trilateration technique given in Reference 2. The initial estimate for each succeeding observation point is taken from the results of the previous one. From the inertial coordinates of the tracking station location and of the satellite, the program computes a range for each observation point and compares these to the observed ranges. By using a Taylor series expansion and a least squares process a converged set of the inertial coordinates of the satellite is then determined. If there are only three observation points, Cramer's rule may be used to solve the set of three simultaneous equations for the position of the satellite. In addition, the multilateration program referred to in this report may be used to obtain a solution if there are more than three observation points.



II. DATA TYPES

Two types of data were used. The two way delay data is the normal kind from the ATSR ranging system. It measures the time it takes for a signal to travel from the station to the satellite and return to the ground. This data type and its measurement is given in references 3 and 4. The other data type is four way delay time or Turnaround Ranging. This measurement signal leaves the master station, travels to the satellite, returns to the slave station, is turned around to the satellite and finally is returned to the master station. The master station must have tracked in the two way mode and in the same time period as well, in order to extract the slave station's range at this time from the turnaround range.

Trilateration Experiment

In order to be able to trilaterate in the sense of the Hughes trilateration program, it is necessary to obtain a series of overlapping range values to three different stations. The Hughes Trilateration program then creates a position and velocity vector from this information. For detailed information about this program see reference 1. One of the functions of the software package described in this document is to create the required format from the polynomials representing the raw data. See Reference 4 for raw data format information.

Turnaround Ranging

From the four way delay data a range to the satellite must be determined. The only station that needs to be fully equipped is the master station. The slave station need only to be able to turn the signal around. The output of this software package is a preprocessed series of ranges to be input to the trilateration program, and a DODS observation tape to be input to an orbit determination program.

III. ALGORITHM AND FORMULAS

This section will describe the formulas used and how they are derived to process this data. The details of the two way range data type are described in reference 3 and it is suggested that the reader read that document. The actual programmed equations will be marked with a starred number. It is hoped that the intermediate steps will make their derivation clear. The major problem in interpreting this data is that the time on the raw data message is not the proper observation time tag. Iterating so as to converge on the proper time tag and relating this time tag to the raw data time is the one complication of the procedure described in this algorithm.

A smoothing program creates a Chebyshev polynomial up to 12th degree of raw data and raw data time. See reference 3, Section 6.3. The spans of data have to be organized and read into this program in the groups that one wishes to have fitted with polynomials. There are sorting programs to do this. The polynomial coefficients and other details for creating the DODS Observation Tape are passed to the multilateration program where the ranges and proper time tags are then determined.

A time T_R, is chosen to be satellite time. The relationship between the raw data time and the ground received time is shown in the following figures. In order to find the proper measurement to correspond to the satellite time T_R, one needs the raw data time T_D that corresponds (the measurement polynomial is a function of raw data time). The connection between the times can be achieved by iterating to find the proper raw data time and therefore proper measurement to correspond to the satellite time, T_R. The convergence is very rapid and few iterations are necessary.

Let

T_D = raw data time

δ_2 = two way delay time raw measurement

D_M = total two way delay time of the master station

δ_4 = four way delay time raw measurement

D_4 = total four way delay time

D_S = total two way delay time of the slave station

N_{AM} = the ambiguity number for the master station

N_{AS} = the ambiguity number for the slave station

ΔA = the size of the ambiguity in time

T_R = the proper data time tag

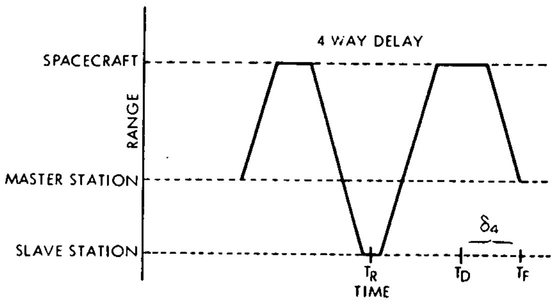
T_I = time the signal left the ground

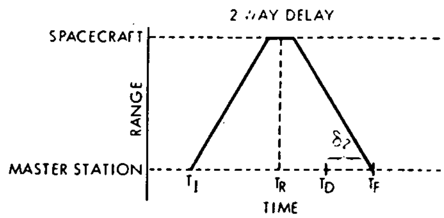
T_F = time the signal returned to the ground

c = speed of light

S_M = master station delay

S_S = slave station delay





One first chooses sets of data which overlap in time. For both trilateration and turnaround ranging it is a requirement to have the ranges from all the stations involved at the same time.

1* T_R = first overlap time

Since the polynomials are in terms of T_D time an estimate of the T_D corresponding to T_R needs to be made. We will first consider the 2 way delay measurement.

Since (1) $$T_R = \frac{T_I + T_F}{2}$$

(2) $$T_I = T_D - N_{AM} \Delta A$$

(3) $$T_F = T_D + \delta_2$$

(4) $$T_R = T_D - \frac{N_{AM} \Delta A}{2} + \frac{\delta_2}{2}$$

(5) $$T_D = T_R + \frac{N_{AM} \Delta A}{2} - \frac{\delta_2}{2}$$

and since δ_2 is unknown at this point the first approximation to T_D is

2* $$T_D = T_R + \frac{N_{AM} \Delta A}{2}$$ from equation (5)

3* Then obtain δ_2 at T_D from the Chebyshev polynomial.

Compute from equation (4)

4* $$T_R' = T_D - \frac{N_{AM} \Delta A}{2} + \frac{\delta_2}{2}$$

Compare

5* If $|T_R/T_R' - 1| > \epsilon$, T_R and T_R' have not converged. According to equation (5) T_D should be

$$T_D = T_R + \frac{N_{AM} \Delta A}{2} - \frac{\delta_2}{2}$$

and we approximated it by

2* $$T_D = T_R + \frac{N_{AM} \Delta A}{2}$$

so let

6* $$T_D = T_D - \frac{\delta_2}{2}$$

and try again from equation 3*.

5* If $|T_R/ T_R' - 1| \leq \epsilon$ T_R and T_R' have converged, then compute

7* $D_M = \delta_2 + N_{AM} \Delta A - S_M$ the master station total delay.

If all the data is two way data one would repeat this process with each station.

At this point, one would have the proper measurement from each station at the same time. One could solve for a position vector at time T_R (trilaterate) if one has ranges from three stations.

If some of the data is four way (turnaround) data the following algorithm must be used.

The four way data contains the sum of ranges to two stations. It is necessary to extract the range to the slave station from this data. In order to do this, one must have tracked the satellite from the master station alone at approximately the same time. This is only necessary because the orbit determination programs currently do not accept the summed range as a data type. Due to this restriction, one relies on the smoothing polynomial to fill in the gaps in the data. It is assumed at this point that the master station total delay is known at time T_R, as obtained in the previous algorithm. It is now necessary to find the four way total delay, D_4 and then to compute the total two way delay for the slave station ($D_S = D_4 - D_M$).

Choose the same T_R as before.

Since (6) $$T_R = \frac{T_I + T_F}{2}$$

(7) $$T_I = T_D - N_{AM}\Delta A - N_{AS}\Delta A$$

(8) $$T_F = T_D + \delta_4$$

(9) $$T_R = T_D - \left(\frac{N_{AM}\Delta A + N_{AS}\Delta A}{2}\right) + \frac{\delta_4}{2}$$

(10) $$T_D = T_R + \left(\frac{N_{AM}\Delta A + N_{AS}\Delta A}{2}\right) - \frac{\delta_4}{2}$$

and since δ_4 is unknown at this point the first approximation to T_D is

8* $$T_D = T_R + \left(\frac{N_{AM}\Delta A + N_{AS}\Delta A}{2}\right).$$

9* Then obtain δ_4 at T_D from the Chebyshev polynomial.

Compare δ_4 and δ_2.

If $\delta_4 \leq \delta_2$ then an extra ambiguity had to occur because of the combined effect of two stations. This means equation (10) needs an extra $\Delta A/2$ added to it and

$$\text{(11)} \qquad T_D = T_R + \left(\frac{N_{AM}\Delta A + N_{AS}\Delta A + \Delta A}{2}\right) - \frac{\delta_4}{2}$$

for this case.

Therefore

$$10^* \qquad T_D = T_D + \frac{\Delta A}{2}$$

and since the initial T_D had to be far off another 10* δ_4 is obtained from the Chebyshev polynomial at the new T_D.

D_4 is then computed.

$$11^* \qquad D_4 = \delta_4 + N_{AM}\Delta A + N_{AS}\Delta A + \Delta A - S_M - S_S$$

$$12^* \qquad T_R' = T_D - \left(\frac{N_{AM}\ \Delta\ A + N_{AS}\ \Delta\ A + \Delta\ A}{2}\right) + \frac{\delta_4}{2} \text{ from equation (11)}$$

If the new $\delta_4 > \delta_2$ recompute T_R and D_4 as if that were the original situation as below. When $\delta_4 > \delta_2$ compute

$$13^* \qquad T_R' = T_D - \left(\frac{N_{AM}\Delta A + N_{AS}\Delta A}{2}\right) + \frac{\delta_4}{2} \quad \text{from equation (9)}$$

and 14* $$D_4 = \delta_4 + (N_{AM}\Delta A + N_{AS}\Delta A) - S_M - S_S$$

If 15* $|T_R/T_R' - 1| \leq \epsilon$, T_R and T_R' have converged then

$$16^* \qquad Ds = D_4 - D_M$$

If 15* $|T_R/T_R' - 1| > \epsilon$, T_R and T_R' have not converged then

let 16* $$T_D = T_D - \frac{\delta_4}{2}$$

and try again from equation 3*.

After two way delays are determined for each station the range R for each station is computed

17* $$R = \frac{c}{2} D, \text{ where } D = D_M \text{ or } D_S$$

The output is properly formatted and the data is time tagged, thus completing this task.

IV. COMPUTATION OF THE INERTIAL COORDINATES OF THE TRACKING STATIONS

From the results of the previous section, we now have a set of ranges (three or more) at the same time. The function of the multilateration program is to compute the sub-satellite point at this time.

The multilateration method takes the set of three or more simultaneous ranges, uses a least squares solution to compute the inertial satellite position, and then computes the sub-satellite point in the earth fixed system. These items are computed according to the formulas presented in sections IV, V, and VI.

However, first one needs to know the coordinates of the ranging stations. The method for computing the inertial coordinates of the tracking stations is the one presented on pages 16-18 of Reference 5.

The following data are needed to compute the coordinates of the tracking stations:

(λ_0) = the hour angle of the first point of Aries

(λ_E) = the geodetic longitude of the terrestrial tracking station in radians, as measured eastward from Greenwich (a negative sign must be prefixed if measured westward from Greenwich)

(θ_D) = the geodetic latitude of the station in radians, measured as positive north of the Equator, and as negative south of the Equator

(H') = the altitude of the station in feet, measured positive above sea level and negative below sea level

(ω) = the angular velocity of rotation of the earth in radians per hour

(ΔT) = the difference in hours between the observation time and midnight preceding the observation time

(f) = the flattening coefficient of the earth

(θ_G) = the geocentric latitude of the tracking station in radians

$(\hat{\rho})$ = the geocentric distance of the tracking station, in units of earth radii

(e) = the eccentricity of the earth

The geodetic longitude (λ_E), geodetic latitude (θ_D), and height (H') of the tracking station are known. From these the inertial geocentric coordinates of the tracking station in spherical coordinates $(\hat{\rho}, \theta_G, \delta)$ can be computed. These spherical coordinates can then be converted to a Cartesian system of coordinates (x_T, y_T, z_T).

In the meridian section of the earth through an observer, the position of the latter relative to the earth's center can be expressed in rectangular coordinates as:

$$\hat{\rho} \sin\theta_G = S \sin\theta_D, \quad \hat{\rho}\cos\theta_G = C\cos\theta_D.$$

These serve to define the auxiliary functions S and C.

$$C = [\cos^2\theta_D + (1-f)^2 \sin^2\theta_D]^{-1/2}$$

$$S = (1-f)^2 C$$

$$H = (4.77865 \times 10^{-8})\,H' \text{ (converts feet to earth radii)}$$

$$\theta_G = \arctan\left[\left(\frac{S+H}{C+H}\right)\right]\tan\theta_D$$

$$\hat{\rho} = [(S+H)^2 \sin^2\theta_D + (C+H)^2\cos^2\theta_D]^{1/2}$$

$$\delta = \lambda_0 + \omega(\Delta T) + \lambda_E$$

(δ) = the angle in radians between the vernal equinox and the observation meridian plane

$$x_T = \hat{\rho} \cos \theta_G \cos \delta$$

$$y_T = \hat{\rho} \cos \theta_G \sin \delta$$

$$z_T = \hat{\rho} \sin \theta_G$$

(x_T, y_T, z_T) = the geocentric coordinates of the tracking station in units of earth radii

We now have the station locations in quantities and units that we can use in the program.

V. COMPUTATION OF THE INERTIAL COORDINATES OF THE SATELLITE

This multilateration method computes the inertial coordinates of the satellite by a least squares iteration procedure, given the ranges from the satellite to three or more tracking stations, the inertial coordinates of the tracking stations, and an initial estimate of the inertial coordinates of the satellite. This section describes the mathematical method for computing the inertial coordinates of the satellite. We refer to this method as the multilateration method.

(R_o) = observed range in earth radii

(R_C) = computed range in earth radii

(x, y, z) = the present estimate of the inertial coordinates of the satellite, obtained initially by using the trilateration method given in Reference 2.

(x', y', z') = the new estimate of (x, y, z)

$$R_C^2 = (x - x_T)^2 + (y - y_T)^2 + (z - z_T)^2$$

A function of three variables may be expanded in a series by Taylor's formula in the form:

$$f(x + \Delta x, y + \Delta y, z + \Delta z) - f(x, y, z)$$

$$= \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y + \frac{\partial f}{\partial z} \Delta z$$

$$+ \text{higher order terms}$$

or $$\Delta f = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z,$$ ignoring the higher order terms

since $$R_C^2 = f(x, y, z)$$

we can write $$\Delta R^2 = \frac{\partial R_C^2}{\partial x}\Delta x + \frac{\partial R_C^2}{\partial y}\Delta y + \frac{\partial R_C^2}{\partial z}\Delta z$$

where $$\Delta R^2 = R_O^2 - R_C^2$$

$$\frac{\partial R_C^2}{\partial x} = 2x - 2x_T$$

$$\frac{\partial R_C^2}{\partial y} = 2y - 2y_T$$

$$\frac{\partial R_C^2}{\partial z} = 2z - 2z_T .$$

Therefore $$\Delta R^2 = (2x - 2x_T)\Delta x + (2y - 2y_T)\Delta y + (2z - 2z_T)\Delta z$$

or $$\frac{1}{2}\Delta R^2 = (x - x_T)\Delta x + (y - y_T)\Delta y + (z - z_T)\Delta z \qquad (5.1)$$

A least squares routine is then used to compute Δx, Δy, Δz. The least squares method used is the one described on pages 61-66 of Reference 5.

Then

$$x' = x + \Delta x$$

$$y' = y + \Delta y$$

$$z' = z + \Delta z$$

The program then returns to Equation 5.1 with x, y, z being replaced by x', y', and z'. ΔR^2 is re-evaluated. This iterative procedure is continued until the standard deviation of fit of $1/2\ \Delta R^2$ reaches the desired tolerance, and we thus have the inertial position of the spacecraft.

VI. COMPUTATION OF THE LONGITUDE AND LATITUDE OF THE SUB-SATELLITE POINT

The method for transforming the inertial position coordinates (x, y, z) of the satellite, as computed above, to the East longitude (λ_s), the geodetic latitude (θ_D), and the elevation of the satellite (H_S) above and normal to the adopted ellipsoid, and the geocentric radius magnitude (ρ) is given in this section.

The symbols used in this section are the same as those used in Section IV, except they refer to the sub-satellite point instead of the tracking station location. The method used in this section may be found in Reference 1.

The converged values of the geocentric coordinates of the satellite (x, y, z) have been found using Section V. Proceeding from these knowns, we compute the unknown longitude, latitude, and elevation as follows:

$$\rho^2 = x^2 + y^2 + z^2$$

$$e^2 = 2f - f^2$$

$$\cos \theta_G = \left[\frac{(x^2 + y^2)}{\rho^2}\right]^{1/2}$$

$$\sin \theta_G = \frac{z}{\rho}$$

$$\tan \theta_G = \frac{\sin \theta_G}{\cos \theta_G}$$

$$\tan \theta_G = (1 - f)^2 \tan \theta_D$$

$$\theta_D = \arctan \left[\frac{\tan \theta_G}{(1 - f)^2}\right]$$

$$\cos \delta = \frac{x}{\rho \cos \theta_G}$$

$$\sin \delta = \frac{y}{\rho \cos \theta_G}$$

$$\lambda_s = \delta - \lambda_0 - \omega(\Delta T)$$

if λ_s is greater than 180°, the program will take λ_s as

$$\lambda_S = \lambda_S - 360°$$

$$\Delta\theta_G' = 0$$

$$\theta_G' = \theta_G - \Delta\theta_G' \qquad (6.1)$$

$$r_c = \left[\frac{1 - e^2}{1 - e^2 \cos^2 \theta_G'}\right]^{1/2}$$

$$\theta_D = \tan^{-1}\left[\frac{\tan\theta_G'}{(1-f)^2}\right]$$

$$H = [\rho^2 - r_c^2 \sin^2(\theta_G - \theta_G')]^{1/2} - r_c \cos(\theta_G - \theta_G')$$

$$\Delta\theta_G' = \sin^{-1}\left[\frac{H}{\rho}\sin(\theta_G - \theta_G')\right]$$

If $\Delta\theta_G'$ has not stopped varying return to Equation (6.1).

$$H_S = a_e H$$

(λ_S) = the geodetic longitude of the sub-satellite point (+ indicates east of Greenwich and - west of Greenwich)

(θ_D) = the geodetic latitude of the sub-satellite point (+ indicates north of the Equator and - south of the Equator)

(a_e) = radius of the earth in kilometers

(H_S) = the height of the satellite above and normal to the adopted ellipsoid.

Thus we have computed the longitude and latitude of the sub-satellite point, and the height of the satellite.

VII. RESULTS

Simulated Data

The multilateration program described herein has been tested using simulated data for the ATS-1 satellite. The epoch used was the 4th of April 1969 at 0 hours, 0 minutes, and 0 seconds.

Some of the orbital parameters were:

semi-major axis = 42,166.5 km.

eccentricity = .000229

inclination = 1.4°

period = 23.94 hr.

The test cases listed below were run using the three observations given below.

	Observation Time						
Tracking Station	yr.	mo.	day	hr.	min.	sec.	Range
Mojave, Calif.	69	4	4	0	37	0	38,133.6887 km.
Rosman, N. C.	69	4	4	0	37	0	40,633.4199
Toowoomba, Australia	69	4	4	0	37	0	39,518.6607

The true values of x, y, z as given by the program which simulated the data were:

$$x = 26{,}297.9477 \text{ km.}$$

$$y = 32{,}944.5272$$

$$z = -573.1739$$

The initial values used for the three test cases were:

	x	y	z
Case 1	+28,701.7425 km.	+28,701.7425 km.	-4464.7155 km.
Case 2	+19,134.4950	+22,323.5775	-6314.3833
Case 3	+25,512.6600	+31,890.8250	-574.0348

All the cases converged to the same values:

$$x = 26{,}298.1470 \text{ km.}$$
$$y = 32{,}944.3690$$
$$z = -573.1558$$

If we compare these values to the given values, we get:

$$\Delta x = -200 \text{ m.}$$
$$\Delta y = +158$$
$$\Delta z = -18$$

Where the difference is taken as the given value minus the computed value.

The sub-satellite point is then found to be:

$$\begin{aligned} \text{Longitude} &= -141.66^\circ \\ \text{Latitude} &= -0.784^\circ \\ \text{Height of satellite} &= 35{,}779.31 \text{ km.} \end{aligned}$$

The program gave similar results when Cramer's method was used instead of a least squares procedure. For example, using Cramer's method for Case 1 gave the following results:

$$\begin{aligned} x &= 26{,}298.1546 \text{ km.} \\ y &= 32{,}944.3582 \\ z &= -573.1764 \end{aligned}$$

$$\begin{aligned} \Delta x &= -207 \text{ m.} \\ \Delta y &= +169 \\ \Delta z &= +2 \end{aligned}$$

$$\begin{aligned} \text{Longitude} &= -141.660^\circ \\ \text{Latitude} &= -0.784^\circ \\ \text{Height of satellite} &= 35{,}779.92 \text{ km.} \end{aligned}$$

The largest error in the test cases was 207 meters. These test cases do not agree exactly because the program generating the simulated data and the multilateration program did not use the same sidereal time. These test cases were run before the trilateration method for computing the initial estimates of x, y, z had been incorporated into the multilateration program. Guided by these results, the program was then run for three cases using real satellite data.

Real Data

Case 4

The observations for this case were taken on October 1, 1969 by three ATS ground stations: Rosman, North Carolina; Mojave, California; and Toowoomba, Australia. Mojave was the prime or two-way delay station; whereas, Toowoomba and Rosman were four-way delay stations. Two-way delay means that the signal is sent from the station to the satellite and is then returned to the station. Four-way delay means that the signal is sent from station A to the spacecraft, from the spacecraft to station B, from station B to the satellite, and from the satellite to station A. Station A would be the master station in this case.

The three stations tracked the spacecraft during a ten minute span of time as illustrated in Figure 2. The data rate was one observation per second and each station ranged for about one minute each time it tracked.

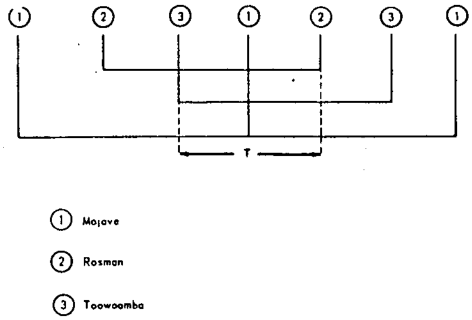


Figure 2.

Some of the orbital parameters used are:

semi-major axis = 42,168.1 km.
eccentricity = .00229
period = 23.94 hr.

The initial values used for the inertial coordinates are:

x = +5740.3485 km.
y = -41,458.0725
z = -3507.9907

The results are then compared to the results obtained by the Refined World Map Program (Reference 6).

	Observation Time			Satellite Position		
Program	hr.	min.	sec.	Long. (+E)	Lat. (+N)	HT
ATS	3	47	56.00	-148.710°	-.484°	35782.0 km.
WMAP	3	48	0.00	-148.650	-.489	35787.0
ATS	3	53	56.01	-148.734	-.536	35780.7
WMAP	3	54	0.00	-148.655	-.537	35788.7
WMAP	4	41	0.00	-148.660	-.896	35789.0
ATS	4	41	35.00	-148.779	-.822	35787.0
WMAP	4	42	0.00	-148.660	-.903	35790.0

Here, ATS refers to the multilateration program developed in this report.

The longitude agrees to within .1°, the latitude within .01° and the height within 10 kilometers. Some of the error is probably due to the delays in the ground equipment at the master station not being measured accurate enough.

Case 5

The observations for this case were taken on December 23, 1969 by the ATS ground stations: Rosman, Mojave, and Toowoomba. This time the stations made simultaneous observations. All the stations were therefore two-way delay stations. They give an initial estimate for x, y, z of

$$x = +21{,}934.0285 \text{ km.}$$
$$y = -35{,}990.6776$$
$$z = -1103.2987$$

Below is the comparison of the results of the multilateration program (called ATS in the charts) with the ORB-1 and WMAP programs (Reference 6).

Program	Observation Time yr.	mo.	day	hr.	min.	sec.	x	y	z
ATS	69	12	23	0	22	0.02	25,677.7275 km.	-33,421.9067 km.	-1124.1973 km.
ORB-1	69	12	23	0	22	0.00	25,683.7708	-33416.8086	-1125.5546
ATS	69	12	23	0	23	1.02	25,826,0626	-33,307.2853	-1128.8897
ORB-1	69	12	23	0	23	0.00	25,829.6496	-33,304.0089	-1130.0527

	Observation Time						Satellite Position		
Program	yr.	mo.	day	hr.	min.	sec.	Long. (+E)	Lat.(+N)	Ht
ATS	69	12	23	0	22	0.02	-149.340°	-1.538°	35783.8 km.
WMAP	69	12	23	0	22	0.00	-149.327	-1.531	35784.0
ATS	69	12	23	0	23	1.02	-149.340	-1.545	35783.8
WMAP	69	12	23	0	23	0.00	-149.327	-1.537	35784.0

The longitude agrees to within .1° the latitude within .01°, and the height within 1 kilometer.

Case 6

The observations for this case were taken on February 20 and 21, 1970. The observations were taken simultaneously by the three tracking stations of Mojave, Kashima, and Toowomba.

Below is a comparison of the multilateration program (called ATS in the charts), with the TV-1, ORB-1, and WMAP programs.

		Observation Time					Satellite Position		
Program	yr.	mo.	day	hr.	min.	sec.	x	y	z
ATS	70	2	20	4	40	30.26	+14843.66km.	+39476.84km.	-349.72km.
TV-1	70	2	20	4	40	30.26	+14844.79	+39476.42	-349.72
ORB-1	70	2	20	4	40	30.00	+14844.87	+39476.33	-350.27
ATS	70	2	20	10	40	29.26	-39487.29	+14695.89	+1642.31
TV-1	70	2	20	10	40	29.26	-39486.87	+14702.83	+1642.26
ORB-1	70	2	20	10	40	30.00	-39487.87	+14699.94	+1641.75
ATS	70	2	20	17	10	30.25	-9206.24	-41136.44	+120.00
TV-1	70	2	20	17	10	30.25	-9207.42	-41136.18	+120.00
ORB-1	70	2	20	17	10	30.00	-9207.20	-41136.14	+119.27
ATS	70	2	20	21	10	28.28	+31088.54	-28439.97	-1391.48
TV-1	70	2	20	21	10	29.29	+31087.26	-28438.56	-1391.55
ORB-1	70	2	20	21	10	30.00	+31096.58	-28436.10	-1392.39

		Observation Time					Satellite Position		
Program	yr.	mo.	day	hr.	min.	sec.	Long. (+E)	Lat. (+N)	Ht.
ATS	70	2	20	4	40	30.26	-150.44°	- .48°	35798.6 km.
TV-1	70	2	20	4	40	30.26	-150.44	- .48	35798.6
WMAP	70	2	20	4	40	30.00	-150.43	- .47	35798.4
ATS	70	2	20	10	40	29.26	-150.49	+2.23	35789.2
TV-1	70	2	20	10	40	29.26	-150.49	+2.23	35789.2
WMAP	70	2	20	10	40	30.00	-150.49	+2.23	35789.0
ATS	70	2	20	17	10	30.25	-150.46	+0.16	35776.0
TV-1	70	2	20	17	10	30.25	-150.46	+0.16	35776.0
WMAP	70	2	20	17	10	30.00	-150.45	+0.16	35775.8
ATS	70	2	20	21	10	28.28	-150.45	-1.89	35778.5
TV-1	70	2	20	21	10	29.29	-150.45	-1.89	35778.5
WMAP	70	2	20	21	10	30.00	-150.45	-1.89	35778.4

The longitude agrees to within 0.1°, the latitude within .01°, and the height within 1 kilometer. The TV-1 and the multilateration programs did not give the same results, because the TV-1 programs used mean sidereal time and the multilateration program used apparent sidereal time.

From these results it can be seen that there is excellent agreement between the various position determination methods. The trilateration method has advantages over conventional orbit determination techniques in that it is a geometric solution and does not require a sophisticated force model, nor does it require a number of iterations to obtain an orbit as do conventional orbit determination techniques. Each position vector takes less than five seconds CPU time on the 360/91 when using the trilateration program. The time required by conventional orbit determination programs depends on the length of the arc needed to determine the orbit, the number of iterations necessary to converge, and the force model used. Some disadvantages of the described trilateration technique are; an orbit is not determined and therefore orbit prediction can not be made, and there must be mutual visibility by three stations capable of tracking almost simultaneously the particular satellite. For additional information or the future development of the trilateration technique the reader should refer to references 8 and 9.

VIII. CONCLUSION

From the results of the cases given in the previous section, it can be seen that the multilateration programsgives good results. The Multilateration, TV-1, ORB-1 and WMAP programs compared very favorably. The longitude always agreed to within .1 degrees, and the latitude always agreed to within .01 degrees. The height agreed to within 1 kilometer, except on the turnaround ranging case. A possible explanation for this discrepancy is that the measurements of the time delay in the ground equipment at the slave station is inaccurate. The results further indicate that spacecraft positions for synchronous satellites can be obtained using less computer time and less computer memory if one uses the multilateratinn instead of using conventional orbit determination techniques.

IX. ACKNOWLEDGMENT

The authors would like to express their appreciation to N. L. Bonavito, Dave Stewart, and Harvey Walden of the Trajectory and Dynamics Branch for their advice in the development of the multilateration program and this report.

X. REFERENCES

1. Escobal, P. R., and Lange, D. M., "Trilatron (TV-1) Operational Trilateration Program Version 1 for Satellite Position Determination," Hughes Aircraft Company, NASA Contract NAS5-3823.

2. Escobal, P. R., "Methods of Orbit Determination," John Wiley and Sons, New York, July 1965.

3. Grenchik, Tom and Putney, Barbara, "A Review of Goddard Range and Range Rate System Measurements and Data Processing Techniques," Goddard X Document X-551-69-137, April 1969.

4. Zillig, Dave, "Data Formats of the Goddard Range and Range Rate System and the Application Technological Satellite Range and Range Rate System," Goddard X Document X-571-69-149, April 1969.

5. Walden, Harvey, "Orbital Prediction and Differential Correction Using Vinti's Spheroidal Theory for artificial Satellites," NASA Technical Note D-3803, March 1967.

6. Jones, Rosemary C., "AOPB Systems Manual, Program Description R104 Master Orbit Tape Routine," Goddard X Document X-542-66-291, February 1966.

7. Escobal, P. R., Cox, R., and Anzel, B. M., "Trilateration State Determination and Propagation Accuracy Study for Synchronous Satellites," Hughes Aircraft Company, Data Bank Report No. 1-4235, NASA Contract NAS-5-21554, December 1971.

8. "Explanatory Supplement to the Astronomical Ephemeris and The American Ephemeris and Nautical Almanac," prepared by the Nautical Almanac Offices of the United Kingdom and the United States of America, London: H. M. Stationery Office, 1961.

9. Escobal, P. R., Lange, D. M., and Johnson, R. H., "Trilateration and Position Prediction," Hughes Aircraft Company, NASA Contract NAS-5-21554, December 1971.

APPENDIX

Program Information

A. Operating Instructions for Multilateration Program

Below is an explanation of the data cards, with some typical values in parenthesis

1. Card #1 - 5 variables in the format (5E15.8)

   a. f = the flattening coefficient of the earth (.33528919)

   b. TOL = the tolerance that is used to determine when the least squares program has converged to a solution ($.1 \times 10^{-8}$)

   c. DELA = the size of the ambiguity in time in seconds (.125)

   d. REARTH = the radius of the earth in kilometers (6378.165)

   e. C = the speed of light in km./sec. (2.997925×10^{5})

2. Card #2 - 7 variables in the format (2(I5, 5X), F5.1, I5, 5X, E15.8, 2I5)

   a. NOOBS = the number of tracking stations being used in this run. The program with slight modification will handle up to 10 stations. (3)

   b. NOPRST = the station number of the prime or two way delay station. If all stations are two way delay stations this number would be set to -1. (47)

   c. DELTAT = the time increment in minutes at which the data points will be printed out in each time interval. (1)

   d. NOPTS = the number of data points that are to be computed in each time interval. This option would override the value of DELTAT. This option is usually not used and the value is set to -1. (-1)

e. EPSLON = the tolerance which is used to determine when the program has converged on the proper time (1.0×10^{-18}).

f. IMON = the month in which the observations begin (10).

g. IDAY = the day of the month on which the observations begin (1)

3. Card #3 to card # (2 + NOOBS) - 3 variables in the format (2I5, 5X, E15.8))

   a. NUMST = the number of the tracking station (47).

   b. NA = the number of ambiguities (2)

   c. STNDEL = the station delay time in micro-seconds (1.75).

4. Card #(2 + NOOBS) to Card # (32 + NOOBS) - 1 variable in the format (7X, E15.8). 30 cards total

   a. ALAMDM = the hour angle of the first point of Aries at midnight Greenwich mean time in radians. The first card in this set contains the hour angle for the IMON and IDAY on card #2. Each subsequent card has the hour angle for the next day. As the program is presently set up 30 cards are required, however if there is data only on one day only the first card need be filled in the other 20 may be blank. If NOOBS on card #2 is 2 or less all 30 cards may be blank. (.16668687, value for Oct. 1, 1969, obtained from Reference 8.)

   Last 4 cards give the coordinates of the four ATS tracking stations in the format (13X, E15.8, 1X, E15.8, 1X, F8.1).

   a. ALAMDE = the geodetic longitude of the tracking station in radians as measured positive eastward from Greenwich (+4.2430894)

   b. THETAD = the geodetic latitude of the tracking station in radians as measured positive north of the equator (+.61665814).

   c. ALT = the altitude of the tracking station in feet measured as positive above sea level (+3072.8).

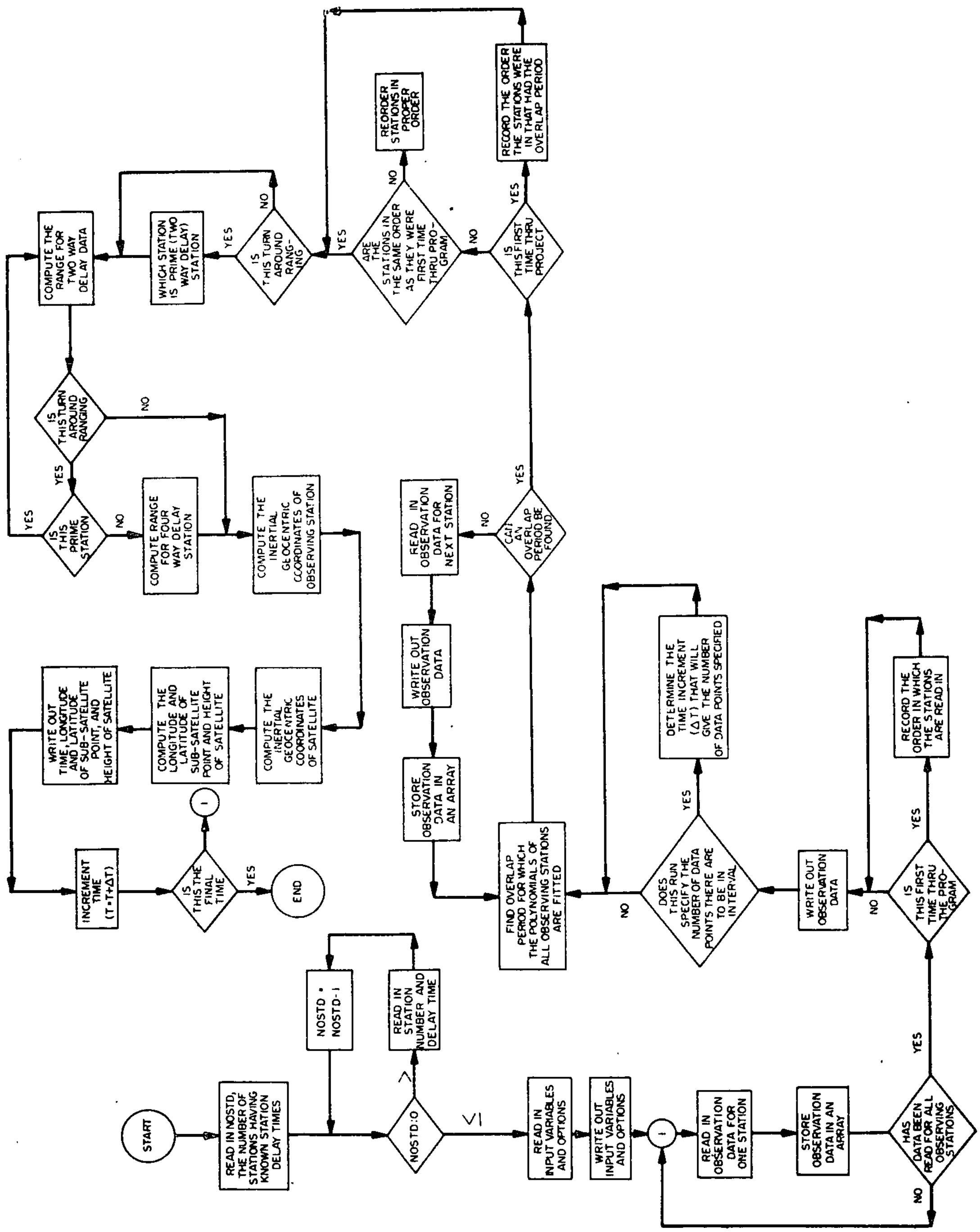
   Note: The name of the station is given in columns 3-8 (Mojave in the example given in a thru c above), but is not used in the program.

B. Program Flow Chart

The program is presently set up for the ATS satellites and must have the four tracking stations in the order given below.

1. Kashima, Japan
2. Mojave, California
3. Rosman, North Carolina
4. Toowoomba, Australia

If any other stations but these four stations are used the multilateration program has to be slightly modified, because the station numbers are hard coded in the program.


START
READ IN NOSTD, THE NUMBER OF STATIONS HAVING KNOWN STATION DELAY TIMES
NOSTD:0
>
≤
READ IN STATION NUMBER AND DELAY TIME
NOSTD = NOSTD-1
READ IN INPUT VARIABLES AND OPTIONS
WRITE OUT INPUT VARIABLES AND OPTIONS
1
READ IN OBSERVATION DATA FOR ONE STATION
STORE OBSERVATION DATA IN AN ARRAY
HAS DATA BEEN READ FOR ALL OBSERVING STATIONS
NO
YES
IS THIS FIRST TIME THRU THE PROGRAM
YES
RECORD THE ORDER IN WHICH THE STATIONS ARE READ IN
NO
WRITE OUT OBSERVATION DATA
DOES THIS RUN SPECIFY THE NUMBER OF DATA POINTS THERE ARE TO BE IN INTERVAL
YES
DETERMINE THE TIME INCREMENT (ΔT) THAT WILL GIVE THE NUMBER OF DATA POINTS SPECIFIED
NO
FIND OVERLAP PERIOD FOR WHICH THE POLYNOMIALS OF ALL OBSERVING STATIONS ARE FITTED
CAN AN OVERLAP PERIOD BE FOUND
NO
READ IN OBSERVATION DATA FOR NEXT STATION
WRITE OUT OBSERVATION DATA
STORE OBSERVATION DATA IN AN ARRAY
YES
IS THIS FIRST TIME THRU PROJECT
YES
RECORD THE ORDER THE STATIONS WERE IN THAT HAD THE OVERLAP PERIOD
NO
ARE THE STATIONS IN THE SAME ORDER AS THEY WERE FIRST TIME THRU PROGRAM
NO
REORDER STATIONS IN PROPER ORDER
YES
IS THIS TURN AROUND RANGING
YES
WHICH STATION IS PRIME (TWO WAY DELAY) STATION
NO
COMPUTE THE RANGE FOR TWO WAY DELAY DATA
IS THIS TURN AROUND RANGING
NO
YES
IS THIS PRIME STATION
YES
NO
COMPUTE RANGE FOR FOUR WAY DELAY STATION
COMPUTE THE INERTIAL GEOCENTRIC COORDINATES OF OBSERVING STATION
COMPUTE THE INERTIAL GEOCENTRIC COORDINATES OF SATELLITE
COMPUTE THE LONGITUDE AND LATITUDE OF SUB-SATELLITE POINT AND HEIGHT OF SATELLITE
WRITE OUT TIME, LONGITUDE AND LATITUDE OF SUB-SATELLITE POINT, AND HEIGHT OF SATELLITE
INCREMENT TIME (T=T+ΔT)
IS THIS THE FINAL TIME
YES
END
1

C. Multilateration Program

```
      IMPLICIT REAL*8(A-H,O-Z,$)
      REAL MAXR,MAXRD,MAXXY,MAX,MIN
      INTEGER CC1,CC2,CC3,CC4,C1SAV,C2SAV,C3SAV,C4SAV,U,V
      REAL*8 OTIME,STIO,OBSV,TRANOM,FICRP,OBCV
      INTEGER*4 ISATN,IOBN
      REAL*4 OBWT,COC,POC,SDFT,FNRRES,TMER,TCRV,ROBWT,RROBWT,AOBWT
      INTEGER*2 IESN,IDRPN,IOT,IEF,ITTI,ITCI,IOCI
      DIMENSION A(10,13), AC(13), ALAMCM(30), CLMQI(10,3), COEFF(13),
     1DELAY(10), DELR(10,1), CLTAQI(3,4), END(10), NDAY(12), NDEG(10),
     2NCMST(10), R(90), RCBS(10), STACCN(4,3), START(10), TYPE(10),
     3WT(10), XT(10), YT(10), ZT(10), RTV1(200, 3), TVTIME(200, 3),
     4I11(2), STATNM(10), STDFIT(10), WTOB(10), DELOUT(200,10),
     5NSTAT(10), STOR(13), STADEL(100), DELDOD(200,10), NAMB(100)
      DIMENSION NAMBPR(10)
      EQUIVALENCE (FIDRP,I11)
      NAMELIST/NAM1/N, NUMST, Y, NAMB, DELA, STNDEL, DEL2W, D2WAY, DELAY
     1 , Y2, Y4
      NAMELIST/NAM2/N, NUMST, Y, NAMB, DELA, STNDEL, DEL2W, D2WAY, DELAY
     1 , Y2, Y4
      NAMELIST/NAM3/N, NUMST, Y, NAMB, DELA, STNDEL, DEL2W, D2WAY, DELAY
     1 , Y2, Y4
C     CORRECTION FOR LIGHT TIME AND AMBIGUITY REQUESTED
      INTEGER*2 IAMBRS
      DIMENSION IAMBRS(3)
      DATA IAMBRS/16,22,31/
C     RANGE RR ANGLES CORRECTICNS TO BE MADE
      DATA TRANOM,FIDRP,OBSV,CBWT,COC,PCC,FNRRES,TMER,TCRV,IESN,IDRPN,
     1IOT,IEF,ITTI,ITCI,IOCI/9*0.,7*0/
  196 IOT      = 1
      AE       = 6378.165
      RADDEG   = 57.2957795
      NDAY(1)  = 31
      NDAY(2)  = 28
      NDAY(3)  = 31
      NDAY(4)  = 30
      NDAY(5)  = 31
      NDAY(6)  = 30
      NDAY(7)  = 31
      NDAY(8)  = 31
      NDAY(9)  = 30
      NDAY(10) = 31
      NDAY(11) = 30
      NDAY(12) = 31
C
C     IF NCPTS = 1 OR > 1,,THE PROGRAM WILL COMPUTE A RANGE FOR THE
C     NOPTS SPECIFIED. NO CCMPUTATICN CF INERTIAL COORDINATES OR
C     OF SUB-SATELLITE POINT WILL BE MADE IN THIS CASE.
C       NCPRST = -1 FOR THIS CASE. SET NOPTS = 0 FOR REGULAR CASE.
C     IN THIS CASE THE PRCGRAM WILL CCMPUTE THE TIME CCMMCN TO ALL
C     STATIONS AND COMPUTE RANGE, AND SUB-SATELLITE POINT FOR THIS TIME
C     AND CONTINUE TO CCMPUTE AT SPECIFIED INTEGRALS OF TIME.
C
      READ(5,199) F, TOL, DELA, REARTH, C, NCOBS, NOPRST, DELTAT, NOPTS,
     1 EPSLCN, IMCN, IDAY
  199 FORMAT(5E15.8/2(I5,5X), F5.1, I5, 5X, E15.8, 2I5)
      WRITE(6,5000) F, TOL, DELA, NOOBS, NOPRST, DELMIN, REARTH, C
      WRITE(6,5019)
 5C19 FORMAT(1H0, 37X, 17HTRACKING STATIONS)
      DO 195 I = 1, NOOBS
      READ(5, 194) NUMST, NA, STNDEL
  194 FORMAT(2I5, 5X, E15.8)
      STNDEL   = STNDEL * 1.0D-06
      WRITE(6,5020) NUMST, NA, STNDEL
```

```
5C20 FORMAT(1HO,
    117H NO. OF STATION =, I5, 5X, 14H NO. OF AMB. = I5, 5X,
    216H STATION DELAY =, E2C.8, 2X, 4HSEC.//)
     STADEL(NUMST) = STNCEL
 195 NAMB(NUMST) = NA
     DELMIN  = DELTAT
C
C     IF NOPRST  = -1, ALL STATIONS ARE TWO WAY DELAY STATIONS
C
      READ(5,200) (ALAMDM(I), I = 1, 30)
  200 FORMAT(7X, E15.8)
C
C     READ IN TRACKING STATION LOCATIONS
C
  201 FORMAT(13X, E15.8, 1X, E15.8, 1X, F8.1)
      READ (5,201) ((STACCN(I,J), J = 1,3), I = 1,4)
      IOBN     = 0
      TEMP2    = DELA / 86400.C
      DELTAT   = DELTAT / 1440.0
      N        = 1
      NOTIM    = 0
      NCITER   = 1
      III      = 0
      NOREAD   = 0
      JJ       = 1
      IF(NOPTS) 3, 3, 4
    3 NK       = NOOBS
      GO TO 5
    4 NK       = 1
    5 DO 10 I = 1, NK
      NOREAD   = NOREAD + 1
      INDIC    = -1
      INDCT    = -1
      READ(19,END=8998) STTIM, FINTIM, NUMST, DATYP, NDGREE, COEFF,
     1ISAT, ROBWT, SDFT, STNAME, FIDRP, IOCI
C
C     THIS MEANS NO GATING IS DONE BY DODS, NOTICE THIS IS GOOD FOR
C     RANGE ONLY, DO NOT USE IF RANGE RATE IS ALSO USED
C
      III(2)  = III(2) + 2
      NOMST(I) = NUMST
      IF(NOREAD - NOOBS) 6, 6, 7
    6 III      = III + 1
      NSTAT(III) = NOMST(I)
    7 START(I) = STTIM
      END(I)   = FINTIM
      TYPE(I)  = CATYP
      NDEG(I)  = NDGREE
      STATNM(I) = STNAME
      STDFIT(I) = SDFT
      WTOB(I) = ROBWT
      KK       = NDEG(I) + 1
      DO 11  J = 1, KK
   11 A(I,J)   = COEFF(J)
   12 IF(J - 13) 13, 10, 10
   13 J = J + 1
      A(I,J) = 0.
      GO TO 12
   10 WRITE(6,5001) I, START(I), END(I), NOMST(I), TYPE(I), NDEG(I),
     1(A(I,J), J = 1, 13)
```

```
5C01 FORMAT(1HO, 3HI = I3//
    110H START    = E20.8//10H END      = E20.8//10H NOMST    = I5//
    210H TYPE     = E20.8//10H NDEG     = I5    //10H A(1)    = E20.8//
    310H A(2)     = E20.8//10H A(3)     = E20.8//10H A(4)     = E20.8//
    410H A(5)     = E20.8//10H A(6)     = E20.8//10H A(7)     = E20.8//
    510H A(8)     = E20.8//10H A(9)     = E20.8//10H A(10)    = E20.8//
    610H A(11)    = E20.8//10H A(12)    = E20.8//10H A(13)    = E20.8//)
     GO TO 215
 139 INDCT    = +1
     IF(NOPTS) 415, 415, 140
 140 M        = 1
     ENDTIM   = FINTIM
     IF(NOPTS - 2) 141, 142, 143 .
 141 T        = STTIM + ((ENDTIM - STTIM) / 2.)
     DELTAT   = ENDTIM
     GO TO 25
 142 DELTAT = (FINTIM - STTIM) / 3.
     DELTAT   = DELTAT - TEMP3
     T        = STTIM + DELTAT
     ENDTIM   = FINTIM - (DELTAT / 2.)
     GO TO 25
 143 DELTAT   = (ENDTIM - STTIM) / (NOPTS - 1.)
     DELTAT   = DELTAT - TEMP3
     T        = STTIM
     GO TO 25
 415 BEGTIM   = START(1)
     ENDTIM   = END(1)
     DO 19  I = 2, NOOBS
     IF(START(I) - BEGTIM) 17, 17, 16
  16 BEGTIM   = START(I)
  17 IF(END(I) - ENDTIM) 18, 19, 19
  18 ENDTIM   = END(I)
  19 CONTINUE
     IF(BEGTIM - ENDTIM) 490, 490, 405
C
C     NO OVERLAP PERIOD FOUND
C
 405 MM       = NOOBS + 1
     INDIC    = + 1
     NOREAD   = NOREAD + 1
     READ(19,END=8998) STTIM, FINTIM, NUMST, DATYP, NDGREE, COEFF,
    1ISAT, ROBWT, SDFT, STNAME, FIDRP, IOCI
C
C     THIS MEANS NO GATING IS DONE BY CODS, NOTICE THIS IS GOOD FOR
C     RANGE ONLY, DO NOT USE IF RANGE RATE IS ALSO USED
C
     I11(2)   = I11(2) + 2
     START(MM) = STTIM
     END(MM)   = FINTIM
     TYPE(MM)  = DATYP
     NOMST(MM) = NUMST
     STATNM(MM)= STNAME
     NDEG(MM)  = NDGREE
     STDFIT(MM)= SDFT
     WTOB(MM)  = ROBWT
     KK        = NDEG(MM) + 1
     DO 451 J = 1, KK
 451 A(MM,J)   = COEFF(J)
 452 IF(J - 13) 453, 450, 450
 453 J         = J + 1
     A(MM,J) = 0.
     GO TO 452
 450 WRITE(6,5001) MM, START(MM), END(MM), NOMST(MM), TYPE(MM),
    1NDEG(MM), (A(MM,J), J = 1, 13)
     DO 435 I = 1, MM
     IF(END(I) - ENDTIM) 431, 431, 435
```

```
  431 START(I) = START(MM)
      END(I)   = END(MM)
      TYPE(I)  = TYPE(MM)
      NOMST(I) = NOMST(MM)
      STATNM(I) = STATNM(MM)
      NDEG(I)  = NDEG(MM)
      STDFIT(I) = STDFIT(MM)
      WTOB(I)  = WTOB(MM)
      DO 432 J = 1, 13
  432 A(I,J)   = A(MM,J)
      GO TO 415
  435 CONTINUE
  490 IF(NOREAD - (2 * NOOBS)) 251, 210, 210
  251 DO 252 III = 1, NOOBS
  252 NSTAT(III) = NOMST(III)
  210 IF(INDIC) 253, 215, 215
  215 DO 250 JJJ = 1, NOOBS
      DO 216 III = 1, NOOBS
      IF(NOMST(III).EQ.NSTAT(JJJ)) GO TO 220
  216 CONTINUE
  220 IF(III.EQ.JJJ) GO TO 250
      TEMP20  =  START(JJJ)
      TEMP21  =    END(JJJ)
      TEMP22  =   TYPE(JJJ)
      TEMP23  =  NOMST(JJJ)
      TEMP24  = STATNM(JJJ)
      TEMP25  =   NDEG(JJJ)
      TEMP26  = STDFIT(JJJ)
      TEMP27  =   WTOB(JJJ)
      DO 270 KKK = 1, 13
  270 STOR(KKK) = A(JJJ, KKK)
      START(JJJ) =  START(III)
        END(JJJ) =    END(III)
       TYPE(JJJ) =   TYPE(III)
      NOMST(JJJ) =  NOMST(III)
     STATNM(JJJ) = STATNM(III)
       NDEG(JJJ) =   NDEG(III)
     STDFIT(JJJ) = STDFIT(III)
       WTOB(JJJ) =   WTOB(III)
     DO 277 KKK = 1, 13
 277 A(JJJ, KKK) = A(III, KKK)
      START(III) = TEMP20
        END(III) = TEMP21
       TYPE(III) = TEMP22
      NOMST(III) = TEMP23
     STATNM(III) = TEMP24
       NDEG(III) = TEMP25
     STDFIT(III) = TEMP26
       WTOB(III) = TEMP27
     DO 276 KKK = 1, 13
 276 A(III, KKK) = STOR(KKK)
 250 CONTINUE
     DO 260 I = 1, NK
 260 WRITE(6,5001) I, START(I), END(I), NOMST(I), TYPE(I), NDEG(I),
    1(A(I,J), J = 1, 13)
     IF(INDCT) 139, 139, 253
 253 T        = BEGTIM
     M        = 1
  14 N        = NOOBS
     IF(NOPRST) 25, 15, 15
  15 IF(NOMST(N) - NOPRST) 20, 25, 20
  20 N        = N - 1
     IF(N) 30, 30, 15
  30 WRITE(6,5002)
```

```
5002 FORMAT(1H0, 50X, 33H NO TWO WAY RANGING STATION FOUND//)
     GO TO 9000
  25 NUMST    = NOMST(N)
     NAMBPR(N) = NAMB(NUMST)
     TEMP3    = NAMB(NUMST) * TEMP2
     TD       = T + .5 * TEMP3
     NUMST    = NOMST(N)
     STNDEL   = STADEL(NUMST)
     KOUNT    = 1
     N2WAY    = N
     GO TO 80
  31 CALL CHEBY(ND, AC, TD, Y, XST, XEND)
     Y        = Y * 1.0D-06
     Y2       = Y
     YSAVE    = Y
     TEMP1    = .5 * Y / 86400.
     DELCOD(M,N) = TEMP1
     TP       = TD - .5 * TEMP3 + TEMP1
     IF(DABS(T / TP - 1.0) - EPSLON) 40, 40, 35
  35 TD       = TD - TEMP1
     GO TO 31
  40 DELAY(N)  = Y + NAMB(NUMST) * DELA - STNDEL
     DEL2W     = Y + NAMB(NUMST) * DELA - STNDEL
     DELCUT(M,N) = Y - STNDEL
     D2WAY    = Y - STNDEL
     WRITE(6,NAM1)
     T        =  TP
     IF(NOPRST) 42, 41, 41
  42 IF(NOPTS) 26, 26, 65
  26 N        = N - 1
     IF(N) 65, 65, 25
  41 N        = NOOBS
  43 IF(NOMST(N) - NOPRST) 50, 45, 50
  45 N        = N - 1
     IF(N) 65, 65, 43
  50 TEMP3    = 0.5 * TEMP2 * (NAMB(NOPRST) + NAMB(NUMST))
     TD       = T + TEMP3
     NUMST    = NOMST(N)
     NAMBPR(N) = NAMB(NUMST)
     STNDEL   = STADEL(NUMST)
     KOUNT    = 2
  80 NSAV     = N
     IF(NOPTS) 77, 77, 76
  76 N        = 1
  77 ND       = NDEG(N)
     XST      = START(N)
     XEND     = END(N)
     KK       = ND + 1
     DO 81 K = 1, KK
  81 AC(K)    = A(N,K)
     N        = NSAV
     GO TO (31, 51), KOUNT
  51 CALL CHEBY(ND, AC, TD, Y, XST, XEND)
     Y        = Y * 1.0D-06
     Y4       = Y
     YSAVE    = Y
     TEMP1    = .5 * Y / 86400.
     DELCOD(M,N) = TEMP1
     IF(Y4 - Y2) 52, 52, 55
  52 TD       = TD + .5 * TEMP2
  53 CALL CHEBY(ND, AC, TD, Y, XST, XEND)
     Y        = Y * 1.0D-06
     YSAVE    = Y
     Y4       = Y
     DELAY(N) = Y + (NAMB(NUMST) + NAMB(NOPRST)) * DELA - STNDEL + DELA
     TEMP1    = .5 * Y / 86400.
     DELCOD(M,N) = TEMP1
     WRITE(6,NAM3)
     IF(Y4 - Y2) 54, 54, 55
```

```
 54 TP       = TD - TEMP3 + TEMP1 - .5 * TEMP2
    GO TO 56
 55 TP       = TD - TEMP3 + TEMP1
    DELAY(N) = Y + (NAMB(NUMST) + NAMB(NOPRST)) * DELA - STNDEL
 56 IF(DABS(T / TP - 1.C) - EPSLON) 6C, 60, 57
 57 TD       = TC - TEMP1
    GO TC 51
 60 DELAY(N) = DELAY(N) - DEL2W
    DELCUT(M,N) = Y - D2WAY - STNDEL
    N        = N - 1
    IF(N) 65, 65, 43
 65 IF(NCPTS) 82, 82, 83
 82 DC 66 N = 1, NOCBS
    R(N)     = DELAY(N) * C / (2.0 * REARTH)
    RTV1(M,N) = R(N)  * REARTH * 100C.0
    TVTIME(M,N) = T
 66 ROBS(N) = R(N) * R(N)
    GO TO 85
 83 R(JJ)    = DELAY(N) * C / (2.0 * REARTH)
    RTV1(M,N) = R(JJ) * REARTH * 100C.0
    TVTIME(M,N) = T
 85 IF(NCPTS) 90, 90, 145
 90 IF(NCOBS - 3) 145, 91, 91
 91 I        = 1
    NLSCIT   = 1
    WT(1)    = - 1.0
    CALL READCL(SYRE, SCNTH, SDAYY, SHOURS, SUETS, SSECCN, KK, T,SE)
    KYR      = SYRE
    KMON     = SONTH
    KDAY     = SDAYY
    KHR      = SHOURS
    KMIN     = SUETS
    SECS     = SSECON
    Y        = YP
202 IF(NCMST(I) - 47) 68, 67, 68
 67 NOSTN    = 2
    GO TO 75
 68 IF(NCMST(I) - 58) 70, 69, 70
 69 NOSTN    = 3
    GO TO 75
 70 IF(NCMST(I) - 66) 72, 71, 72
 71 NOSTN    = 4
    GO TO 75
 72 IF(NCMST(I) - 68) 74, 73, 74
 73 NCSTN    = 1
    GO TO 75
 74 WRITE(6, 5003) NOMST(I)
5C03 FORMAT(1HO, 11HSTATION NO.,2X, I3, 2X, 58HIS NOT INCLUDED IN THE
   1STATION CONSTANTS, PRCGRAM HALTEC.//18X, 65HCHECK THE STATION NO.,
   2OR ACC NEW STATION TO PROGRAM ANC TRY AGAIN)
    GC TC 9C00
 75 MDAY     = KDAY
    IF(KMON - IMON) 1CO, 11C, 1CO
100 MDAY     = MCAY + NDAY(IMON)
110 LROW     = MDAY - IDAY + 1
    ALAMCO   = ALAMDM(LRCW)
    ASECS    = 3600 * KHR + 60 * KMIN
    ASECS    = ASECS + SECS
    DELT     = ASECS / 360C.C+CO
    ALAMCE   = STACON(NOSTN, 1)
    THETAC   = STACON(NOSTN,2)
    ALT      = STACON(NOSTN, 3)
    SNTHEC   = DSIN(THETAD)
    CSTHEC   = DCCS(THETAD)
    CAPC     = CSTHED ** 2 + ((1. - F) * SNTHED) ** 2
    CAPC     = 1.0 / DSQRT(CAPC)
```

```
      CAPS     = CAPC * (1.0 - F) ** 2
      ALT      = ALT * 4.77865C-C8
      TEMP     = (CAPS + ALT) / (CAPC + ALT)
      TEMP     = TEMP * SNTHED / CSTHED
      THETAG   = DATAN(TEMP)
      SNTHEG   = DSIN(THETAG)
      CSTHEG   = DCOS(THETAG)
      DELTA    =ALAMDO + ALAMDE + DELT * 2.6251613330-01
      SNDELT   = DSIN(DELTA)
      CSDELT   = DCOS(DELTA)
      RHOCAP   = (CAPS + ALT) ** 2
      RHOCAP   = RHOCAP * SNTHED ** 2 + ((CAPC + ALT) * CSTHED) ** 2
      RHOCAP   = DSQRT(RHOCAP)
C
C     COMPUTATION OF INERTIAL CCORDINATES OF OBSERVATICN POINT
C
      XM       = RHCCAP * CSTHEG * CSDELT
      YM       = RHOCAP * CSTHEG * SNDELT
      ZM       = RHOCAP * SNTHEG
      XT(I)    = XM
      YT(I)    = YM
      ZT(I)    = ZM
      I        = I + 1
      IF(I - NOOBS) 202, 202, 116
  116 I        = 1
      IF(NOITER - 1) 118, 118, 400
  118 CALL COMP(XT, YT, ZT, RCBS, XOF, YOF, ZOF)
      X        = XCF
      Y        = YOF
      Z        = ZOF
      WRITE(6,5000) F, TOL, DELA, NCOBS, NOPRST, DELMIN, REARTH, C
 5000 FORMAT(1H1, 50X, 25H ATS EXPERIMENTAL RANGING//54X,
     116HINPUT PARAMETERS//4X, 20HEARTH'S FLATTENING = E15.8, 5X,
     223HCONVERGENCE TOLERANCE = E15.8, 5X, 6HDELA = E15.8,
     32X,3HSEC//4X, 27HNO. OF OBSERVING STATIONS = I5, 5X,
     422HNC. CF PRIME STATION = I5, 5X, 8HDELTAT = E15.8, 2X, 4HMINS//
     518H RADIUS OF EARTH = E20.8, 2X, 3HKM., 5X, 16HSPEEC OF LIGHT =
     6E20.8, 2X, 10HKM. / SEC.//)
      WRITE(6,5010) X, Y, Z
 5C10 FORMAT(1HO,
     166X, 3HX = E15.8//1CX,
     259HINITIAL ESTIMATE OF INERTIAL COORDINATES OF SATELLITE   Y =
     3 E15.8//66X, 3HZ = E15.8)
      WRITE(6, 5005)
 5005 FORMAT(1H1, 50X, 6HCUTPLT//
     1 9X, 4HTIME, 26X, 19HSUB-SATELLITE POINT//1X,
     222HYR MO DAY HR MIN   SEC, 6X, 13HLCNGITUDE(+E), 2X,
     324HLATITUDE(+N)  HEIGHT(KM)//)
  400 CLMCI(I,1)  = X - XT(I)
      CLMCI(I,2)  = Y - YT(I)
      CLMQI(I,3)  = Z - ZT(I)
      RCOMP    = CLMQI(I,1) ** 2 + CLMQI(I,2) ** 2 + CLMCI(I,3) ** 2
      RC2      = DSQRT(RCOMP)
      DELR(I, 1) = .5 * (ROBS(I) - RCOMP)
      IF(I - NOOBS) 120, 130, 130
  120 I = I + 1
      GO TO 4C0
  130 CALL GLSP(CLMQI, NOCBS, 3, DELR, 1, DLTAQI, RESID, SUM, WT, 0,
     1STDERR)
      X        = X + DLTAQI(1,1)
      Y        = Y + DLTAQI(2,1)
      Z        = Z + DLTAQI(3,1)
      XS       = X * X
      YS       = Y * Y
      ZS       = Z * Z
      NLSQIT   = NLSQIT + 1
      SUM      = 0.
      DO 3C0   I = 1, NOCBS
```

```
  300 SUM      = SUM + (2.C * CELR(I,1)) ** 2
      FIT      = DSQRT(SUM / NCOBS)
      I   = 1
      IF(NLSQIT - 20) 350, 35C, 8C00
  350 IF(FIT - TOL) 500, 500, 400
C
C     BEGIN CCMPUTATICN OF SUB-SATELLITE POINT
C
  500 RHOS     = XS + YS + ZS
      I        = 0.0
      DTHEG    = 0.0D0
      TES      = F * (2.0DC - F)
      T2       = (1. - F) ** 2
      RHO      = DSQRT(RHOS)
      SNTHEG = Z / RHO
      CSTHEG = DSCRT((XS + YS) / RHOS)
      TEMP1    = RHC * CSTHEG
      CSDELT = X / TEMP1
      SNDELT = Y / TEMP1
      DELTA = DATAN2(SNDELT,CSDELT)
      XLONG = DELTA - ALAMDO -.2625161333D+00 * DELT
      DEL      = CATAN(Z/DSQRT(XS + YS))
  501 THETAG   = DEL - DTHEG
      I        = I + 1
      T10      = DCOS(THETAG)
      T11      = T10 * T10
      RC       = DSQRT((1.CDC - TES) / (1.0DO - TES * T11))
      XLAT     = DATAN(DTAN(THETAG) / T2)
      T12      = XLAT - THETAG
      T13      = DSIN(T12)
      T14      = T13 * T13
      T15      = DCOS(T12)
      HT       = DSCRT(RHOS - RC * RC * T14) - RC * T15
      T20      = HT * T13/ RHO
      DTHEG    = DARSIN(T20)
      IF(I.LT.10) GO TO 5C1
      XLAT     = XLAT * RACDEG
      XLONG    = XLONG * RADDEC
      HT       = HT * AE
  520 IF(DABS(XLONG) - 18C.0) 510, 510, 700
  700 IF(XLONG) 701, 702, 702
  701 XLONG    = XLCNG + 360.0
      GO TO 510
  702 XLONG    = XLONG -36C.0
  510 WRITE(6,5004) KYR, KMCN, KDAY, KHR, KMIN, SECS, XLONG, XLAT, HT
 5C04 FORMAT(1HO,I2, 1X, I2, 2X, I2, 1X, I2, 2X, I2, 2X, F5.2, 5X, F8.3,
     17X, F7.3, 4X, F10.2)
      WRITE(6,9035) X, Y, Z
 9C35 FORMAT(1HO, 4H X = E20.8//4H Y = E20.8//4H Z = E20.8)
  145 T        = T + DELTAT
      NOITER   = NCITER + 1
      YP       = Y
      M        = M + 1
      JJ       = JJ + 1
      IF(T - ENDTIM) 150, 150, 9000
  150 IF(NOPTS) 14, 14, 25
 9C00 IF(NCPTS) 170, 170, 160
  160 N        = N + 1
      JJ       = 1
      IF(N - NOOBS) 5, 5, 165
  165 N        = 1
  170 NTOT     = M - 1
      WRITE(9,1000) NTOT
      WRITE (6,1000)NTOT
 1C00 FORMAT(I5)                                                  9
      M        = 0
```

```
 8999 M        = M + 1
      WRITE(9,1001)(RTV1(M,NN), NN = 1, NOOBS)
      WRITE(6,1001)(RTV1(M,NN), NN = 1, NOOBS)
      WRITE(9,1001)(TVTIME(M,NN), NN = 1, NOOBS)
      WRITE(6,1001)(TVTIME(M,NN), NN = 1, NOOBS)
 1C01 FORMAT(3D25.15)
      IF(M - NTOT) 8999, 9010, 9010
 9C10 DO 9C50 M = 1, NTCT
      DO 9C50 L = 1, NOCBS
C
C     CONVERT RANGE MEASUREMENTS TO DUL UNITS
C
      IOBN     = IOBN + 1
      ROUT     = RTV1(M,L) * 1.0C-07
      TIME     = TVTIME(M,L)
      T30      =  (NAMBPR(L) * DELA) / (2.0DO * 86400.0DO)
      TIME     = TIME + T3C + CELDOD(M,L)
      IF(L.NE.N2WAY) TIME = TIME + T30 - RTV1(M,N2WAY) / (1000.0 * C *
     1                    86400.0)
      CALL READCL(SYRE, SCNTH, SDAYY, SHOURS, SUETS, SSECON, KK, TIME,
     1SE)
      KYR      = SYRE
      KMON     = SCNTH
      KDAY     = SDAYY
      KHR      = SHOURS
      KMIN     = SUETS
      SECS     = SSECON
      IF(CFLOAT(KYR/4) - CFLOAT(KYR) / 4.0D0) 94, 92, 94
   92 NCAY(2) = 29
   94 NODAY    = 0
      LL       = KMON - 1
      DO 95 II = 1, LL
   95 NCDAY    = NCCAY + NCAY(II)
      NODAY    = NODAY + KDAY
      NSJCS    = (KYR - 58) * 365 + ((KYR - 1) / 4 - 14) + NCDAY + 104
      SJDS     = 1C0.D0 * (DFLCAT(NSJDS) + DFLOAT(KHR) / 24.D0
     1           + CFLCAT(KMIN) / 1440.C0 + SECS / 86400.C0)
      OBCV=0.C0
      RCBWT    = WTCB(L)
      STNAME   = STATNM(L)
      SDFT     = STDFIT(L)
 9050 WRITE (29)SJDS,STNAME,RCUT ,TRANCM,FIORP,OBCV,ISAT,IOBN,ROBWT,COC,
     1POC,SDFT,FNRRES,TMER,TCRV,IESN,ICRPN,IOT,IEF,ITTI,ITCI,IOCI,ICCI
      GO TO 5
 8000 WRITE(6,3050)
 3C50 FCRMAT(1HO, 45X, 31H PRCGRAM HALTED, NO CONVERGENCE)
 8998 REWIND 29
      WRITE (6,1300)
 1300 FCRMAT ('1OUT',19X,'STNAME    OBSV',16X,'FEEDBACK   CORR VAL',
     17X,'SAT NO,OB NO, OB WT',8X,'ST. DEV.   CBTP ECFG OCI')
      CO 1302 J=1,32000
      READ (29,END=5210)SJDS,STNAME,ROUT,TRANOM,FIORP,OBCV,ISAT,IOBN,
     1OBWT,COC,POC,SDFT,FNRRES,TMER,TCRV,IESN,IDRPN,ICT,IEF,ITTI,ITCI,
     2IOCI
 1302 WRITE (6,1301)SJDS,STNAME,ROUT,I11(1),I11(2),OBCV,ISAT,IOBN,
     1OBWT,SDFT,ICT,IEF,ICCI
 1301 FORMAT (1X,D20.12,1X,A8,1X,D20.12,1X,2I5,1X,D14.6,1X,I7,    I6,
     11X,C12.4,1X,C12.4,1X,I2,1X,I2,1X,I6)
 5210 STOP
      END
/*
                                                        0583 CARDS
```

```
C
C      SUBROUTINE COMP
C
       SUBROUTINE COMP(XT, YT, ZT, ROBS, XOF, YOF, ZOF)
       IMPLICIT REAL*8(A-H,O-Z,$)
       REAL*8 LAMBDA
       DIMENSION X(3),Y(3),Z(3),XO(2),YO(2),ZO(2),RO(3,2),RO2(2),CHECK(2)
      1, XT(10), YT(10), ZT(10), ROBS(10), R(3,3), RHO(3), R2(3),
      2 RHON(3,3)
       DO 25 I=1,3
       R(1,I) = -XT(I)
       R(2,I)  = - YT(I)
       R(3,I)  = - ZT(I)
       RHO(I)  = DSQRT(ROBS(I))
       X(I)=R(1,I)
       Y(I)=R(2,I)
    25 Z(I)=R(3,I)
       DO 35 I=1,3
    35 R2(I)=DOT(R(1,I),R(1,I))
       E21=.5D0*(RHO(2)**2-RHO(1)**2-(R2(2)-R2(1)))
       E31=.5D0*(RHO(3)**2-RHO(1)**2-(R2(3)-R2(1)))
       DELTA1=(Z(3)-Z(1))*(Y(2)-Y(1))-(Z(2)-Z(1))*(Y(3)-Y(1))
       A=((X(2)-X(1))*(Y(3)-Y(1))-(X(3)-X(1))*(Y(2)-Y(1)))/DELTA1
       B=(E31*(Y(2)-Y(1))-E21*(Y(3)-Y(1)))/DELTA1
       DELTA2=-DELTA1
       C=((X(2)-X(1))*(Z(3)-Z(1))-(X(3)-X(1))*(Z(2)-Z(1)))/DELTA2
       D=(E31*(Z(2)-Z(1))-E21*(Z(3)-Z(1)))/DELTA2
       EPS1=A*A+C*C+1.D0
       EPS2   =2.D0*(A*B+C*D+X(1)+C*Y(1)+A*Z(1))
       EPS3   =B*B+D*D+2.D0*D*Y(1)+2.D0*B*Z(1)+R2(1)-RHO(1)**2
       RAD   =EPS2   **2-4.D0*EPS1*EPS3
       XO(1)=(-EPS2+DSQRT(RAD))/(2.D0*EPS1)
       XO(2)=(-EPS2-DSQRT(RAD))/(2.D0*EPS1)
       DO 30 I=1,2
       YO(I)=C*XO(I)+D
       ZO(I)=A*XO(I)+B
       RO(1,I)=XO(I)
       RO(2,I)=YO(I)
       RO(3,I)=ZO(I)
       XOF=XO(I)
       YOF=YO(I)
       ZOF=ZO(I)
       DO 75 K=1,3
       DO 75 J=1,3
    75 RHON(J,K)=R(J,K)+RO(J,I)
       IF(DOT(RHON,R)/(DSQRT(R2)*RHO(1)).LE.0.D0) RETURN
    30 CONTINUE
       RETURN
       END
CDOTT
       FUNCTION DOT(A,B)
       IMPLICIT REAL*8(A-H,O-Z,$)
       DIMENSION A(3),B(3)
       DOT=0.D0
       DO 1 I=1,3
     1 DOT=DOT+A(I)*B(I)
       RETURN
       END                                                       0058 CARDS
```

```
C
C       SUBROUTINE CHEBY
C
        SUBROUTINE CHEBY(ND,C,XP,Y,XF,XL)
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION C(13)
        X= 2.D0*(XP-XF)/(XL-XF) -1.D0
        T0=1.0D+0
        T1=X
        Y=C(1)*T0+C(2)*T1
        IF (ND.LT.2) RETURN
        NT=ND+1
        DO 10 K=3,NT
        TN=2.0*X*T1-T0
        Y=Y+C(K)*TN
        T0=T1
   10   T1=TN                                                   0019 CARDS
        RETURN
        END

C
C       SUBROUTINE DOT
C
        IMPLICIT REAL*8(A-H,O-Z,$)
        DIMENSION A(3),B(3)
        DOT=0.D0
        DO 1 I=1,3
    1   DOT=DOT+A(I)*B(I)
        RETURN
        END                                                     0013 CARDS
//OBJECT DD *
/*
//OBJECT DD *

C
C       SUBROUTINE READCL
C       SUBROUTINE READCL (SYRE,SONTH,SDAYY,SHOURS,SUETS,SSECON,
        SUBROUTINE READCL (SYRE,SONTH,SDAYY,SHOURS,SUETS,SSECON,
       1KK,SSUNA,SE)                                            FCP70010
        IMPLICIT REAL*8(A-H,O-Z,$)
        DIMENSION YM(28),YJ(28),OM(13)
C             READ CALENDAR DAYS
        DATA OM/31.D0,28.D0,31.D0,30.D0,31.D0,30.D0,31.D0,31.D0,30.D0,
       131.D0,30.D0,31.D0,0.D0/
        DATA YM/58.D0,59.D0,60.D0,61.D0,62.D0,63.D0,64.D0,65.D0,66.D0,
       167.D0,68.D0,69.D0,70.D0,71.D0,72.D0,73.D0,74.D0,75.D0,76.D0,
       277.D0,78.D0,79.D0,80.D0,81.D0,82.D0,83.D0,84.D0,85.D0/
C       SAO MEAN JULIAN DATE
        DATA YJ/36203.D0, 36568.D0,36933.D0,37299.D0,37664.D0,38029.D0,
       138394.D0,38760.D0,39125.D0,39490.D0,39855.D0,40221.D0,40586.D0,
       240951.D0,41316.D0,41682.D0,42047.D0,42412.D0,42777.D0,43143.D0,
       343508.D0,43873.D0,44238.D0,44604.D0,44969.D0,45334.D0,45699.D0,
       446065.D0/                                               FCP70020
        SUNA=SSUNA                                              FCP70030
        E=+1.D0                                                 FCP70040
        I=1                                                     FCP70070
        K=1
   66   IF (YJ(1)-SUNA)67,67,12
   12   WRITE (6,301) SUNA  ,YJ(1),YJ(28)
        STOP
  301   FORMAT (' OUTSIDE CALENDAR RANGE, MJD    =',F20.4,' RANGE =',
       1F20.5,' TO ',F20.5)
```

```
67    IF (YJ(28)-SUNA)12,68,68                                                FCP70080
68    I=2                                                                     FCP70090
71    IF (YJ(I)-SUNA+.5D0)69,70,70                                            FCP70100
69    I=I+1                                                                   FCP70110
      GO TO 71
70    YRE=YM(I-1)                                                             FCP70130
100   SUNM=SUNA-YJ(I-1)                                                       FCP70140
      KSUNM=SUNM                                                              FCP70150
      SUNM1=KSUNM                                                             FCP70160
      IYRE=YRE+.01D0                                                          FCP70170
      IF(MOD(IYRE,4)) 77,78,77
78    OM(2)=29.D0                                                             FCP70190
      GO TO 105
77    OM(2)=28.D0                                                             FCP70210
105   I=1                                                                     FCP70220
      SM=OM(I)+1.D0                                                           FCP70230
73    IF (SM-SUNM)81,81,72                                                    FCP70240
81    I=I+1                                                                   FCP70250
      IF (I-13)210,72,210                                                     FCP70260
210   SM=SM+OM(I)                                                             FCP70270
      GO TO 73
72    SD=SUNM-(SM-OM(I))+1.D0                                                 FCP70290
      ONTH=I                                                                  FCP70300
201   KSD=SD                                                                  FCP70310
      DAC  =  SD
      DAYY=KSD                                                                FCP70320
      SDAC=DAC
      SDAC=IDINT(SDAC)
      DDAC=SDAC
      SD=DAC-DDAC
      IF (SD)220,221,221                                                      FCP70340
220   SD=1.D0+SD                                                              FCP70350
221   HOUR=SD*24.D0                                                           FCP70360
      KHOUR=HOUR                                                              FCP70370
      HOURS=KHOUR                                                             FCP70380
      UE=(HOUR-HOURS              )*60.D0                                     FCP70390
      KUE=UE                                                                  FCP70400
      UETS=KUE                                                                FCP70410
      SECON=(UE-UETS)*60.D0                                                   FCP70420
      IF (ONTH)207,208,207                                                    FCP70430
208   ONTH=12.D0                                                              FCP70440
207   CONTINUE
  99  SYRE=YRE
      SONTH=ONTH
      SDAYY=DAYY
      SHOURS=HOURS
      SUETS=UETS
      SSECON=SECON
      SSUNA=SUNA
      SE=E
      RETURN                                                          FCP71430
      END                                                                 0080 CARDS
C
C     SUBROUTINE GLSP
C
      SUBROUTINE GLSP(A,MM,NN,B,IPP,X,U,SUM,WT,INVRS,STDERR)               005
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A ( 10,3), B ( 10,1), X (3,4), U ( 10,1), SUM (4),
     1 W (3,4), WT ( 10), STDERR (3)
      M=MM                                                                 008
      N=NN                                                                 009
      IP=IPP                                                               010
      INVRSE=INVRS                                                         011
C     AX=B,NORMAL EQUATIONS,A IS RECTANGULAR MATRIX,WITH M ROWS,N COLUMNS  012
C     WX=W1 ,NEW SET OF NORMAL EQUATIONS, W=TR(A)*A ,W1= TR(A)*B..         013
C     TO FORM MATRIX  W ,WITH N ROWS AND N COLUMNS. (TR= TRANSPOSE )       014
```

```
C  WT = -1. MEANS  NO WEIHGTS  IN PROG.  ASSIGN  WEIGHTS = 1.
C  STDERR  = STANDARD ERRORS OF  UNKNOWNS ( X ) .
C  INVRSE = 1. MEANS  STD. ERRORS ARE CALCULATED.
C  INVRSE = 0. MEANS NO CALCULATIONS OF STD. ERRORS OF ( X )*********
      KK=N+1
      IF(M-N) 80,83,81
   81 DO 5 J=1,N
      DO 5 JJ=1,N
      W(J,JJ) = 0.
      IF (WT(1)) 611, 622, 622
C    NO WEIGHTS CASE
 611  DO 51 I= 1,M
 51   W(J,JJ) = W(J,JJ) + A(I,J) * A(I,JJ)
      GO TO 5
C    WEIGHTS ARE PRESENT
 622  DO 52 I=1,M
 52   W(J,JJ) = W(J,JJ) + A(I,J) * A(I,JJ)   * WT(I)
 5    CONTINUE
C     TO FORM  MATRIX W1 , WHICH IS STORED IN N+1 TO N+P COLUMNS OF W.
C     W1 ,WITH N ROWS AND IP COLUMNS.
      IF ( INVRSE ) 201,203,201
 203  KIP = IP
      GO TO 200
 201  KIP = 1
 200  DO 8 K = 1, KIP
      DO 7 J=1,N
      W(J,KK) = 0.
      IF (WT(1)) 612, 623, 623
 612  DO 511 I=1,M
 511  W(J,KK) = W(J,KK)+A(I,J)* B(I,K)
      GO TO 7
 623  DO 522 I= 1,M
 522  W(J,KK) = W(J,KK)+A(I,J)* B(I,K)   * WT(I)
 7    CONTINUE
   8  KK = KK + 1
      GO TO 106
   83 DO 12 I=1,N
      IF (WT(1)) 613, 624, 624
 613  DO 512 J=1,N
 512  W(I,J) = A(I,J)
      GO TO 12
 624  DO 523 J=1,N
 523  W(I,J) = A(I,J) * WT(I)
 12   CONTINUE
      KKK = N+IP
      J=1
      IF ( INVRSE ) 401,403,401
 403  KKK = KKK
      GO TO 400
 401  KKK = KK
 400  DO 13 JJ= KK,KKK
      IF (WT(1)) 614, 625, 625
 614  DO 513 I= 1,N
 513  W(I,JJ) = B(I,J)
      GO TO 13
 625  DO 524 I= 1,N
 524  W(I,JJ) = B(I,J) * WT(I)
   13 J=J+1
 106  IF ( INVRSE ) 71,72,71
 71   K2 = N + 2
      KIP = N + IP
      KK = N + 1
      DO 75 I=1,N
      DO 75 J = K2,KIP
      IF ( J - I - KK ) 73,74,73
```

```
 74  W(I,J) = 1.
     GO TO 75
 73  W(I,J) = 0.
 75  CONTINUE
 72  CALL  TRIANG (W,N,IP, DET )
C    WHERE W IS THE MATRIX WITH N ROWS AND(N+IP)COLUMNS TO BE TRIANGULA
C    RIZED
C    TO TEST THE SINGULARITY OF MATRIX  W., W = TR(A)*A  , W*X = W1.
     IF ( DET ) 82,80,82
  82 CONTINUE
     GO TO 103
  80 SUM(1) = -1.
     GO TO 104
 103 CALL  SOLVE  (W,N,IP,X)
C    WHERE W IS THE TRIANGULARIZED, X IS THE SOLUTION  MATRIX  WITH
C    N ROWS AND  IP  COLUMNS.
C    TO COMPUTE RESIDUAL MATRIX , U = A*X - B ,U IS RESIDUAL MATRX(M,P)
 105 DO 16 I=1,M
     DO 16 K=1,IP
     U(I,K) =0.
     DO 15 J=1,N
  15 U(I,K) = U(I,K) + A(I,J) * X(J,K )
 16  U(I,K) = B(I,K) - U(I,K)
     DO 18 K=1,IP
     SUM(K) = 0.
     DO 18 I=1,M
  18 SUM(K) =SUM(K) +(U(I,K))**2
     IF ( INVRSE ) 722,104,722
 722 DO 723  I = 1,N
     DO  723 J= 2,IP
 723 X(I,J)  = X(I,J) * (SUM(1) / DFLOAT(M-N))
     DO 724 I = 1,N
     STDERR(I) = 0.
     II = I + 1
 724 STDERR(I) = DSQRT( X(I,II))
 104 RETURN
     END
                                                              0116 CARDS

C
C    SUBROUTINE TRIANG
C
     SUBROUTINE TRIANG(A,M,N,DET )
     IMPLICIT REAL*8 (A-H,O-Z)
     DIMENSION A (3,4)
CTRIANG  TO BE USED WITH SUBROUTINE GLSP
C .              WHERE A IS THE MATRIX WITH M ROWS AND M+N COLUMNS TO
C                BE TRIANGULARIZED.
     DET=1.
     MM1=M-1
     MPN=M+N
     DO 5 J=1,MM1
     MAXX = J
     VALUE   = DABS( A(J,J))
     JP1=J+1
     DO 1 K=JP1,M
     IF (VALUE - DABS(A(K,J))) 2, 1, 1
   2 VALUE   = DABS( A(K,J))
     MAXX=K
   1 CONTINUE
     DO 3 L=J,MPN
     TEMP = A(MAXX,L)
     A(MAXX,L) = A(J,L)
   3 A(J,L) = TEMP
     IF ( MAXX-J) 7,6,7
   7 DET  = - DET
```

```
    6  DET = DET*A(J,J)
       ROW = A(J,J)
       DO 4 L=J,MPN
    4  A(J,L) = A(J,L)/ROW
       DO 5 K=JP1,M
       ROW = A(K,J)
       DO 5 L=J,MPN
    5  A(K,L) = A(K,L)-ROW*A(J,L)
       DET = DET*A(M,M)
       ROW = A(M,M)
       DO 9 L = M,MPN
    9  A(M,L) = A(M,L)/ROW
       RETURN
       END
                                                                  0041 CARDS

C
C      SUBROUTINE SOLVE
C
       SUBROUTINE SOLVE(A,M,N,X)
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION  A (3,4), X (3,4)
CSOLVE    TO BE USED WITH SUBROUTINE GLSP
C                    WHERE A IS THE TRIANGULARIZED MATRIX WITH M ROWS AND
C                    M+N COLUMNS.
C                          X IS THE SOLUTION MATRIX WITH M ROWS AND N
C                    COLUMNS.
       MM1=M-1
       DO 2 L=1,N
       MPL=M+L
       DO 2 K=1,MM1
       MMK=M-K
       MMKP1=MMK+1
       X(M,L)=A(M,MPL)/A(M,M)
       SUM=0.
       DO 3 I=MMKP1,M
 3     SUM=SUM+A(MMK,I)*X(I,L)
 2     X(MMK,L)=A(MMK,MPL)-SUM
       RETURN
       END
                                                                  0024 CARDS
```